Reliability Assessment of Power Plant Operations through Boolean Function Expansion and Mean Time to Failure Analysis

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ABSTRACT:
This paper presents an investigation into the reliability assessment of power plant operations using Boolean Function expansion and Mean Time to Failure (MTTF) analysis. The study focuses on a complex system comprising two power generators within a power house. The primary objective of this system is to ensure uninterrupted power supply from the power house to critical consumers via an output main switch. Reliability calculations are conducted considering failure times of various components, including cables, generators, and main switchboards, assuming arbitrary distribution patterns. Additionally, MTTF for the system under an exponential failure time distribution is determined. Graphical representations are employed to illustrate the utility and efficacy of the proposed model.

Keywords: Reliability, M.T.T.F., Boolean Function, exponential distribution.

INTRODUCTION
In contemporary times, the concept of dependability has transcended the realm of abstraction and is now regarded with the same significance as equipment performance. Consequently, the assessment of reliability has become a fundamental prerequisite in all reliability investigations. Nevertheless, it is widely acknowledged that as the complexity of systems increases, the evaluation of dependability becomes disproportionately intricate. Therefore, the ability to derive a concise and simplified symbolic statement regarding reliability for complex systems is paramount.

The challenge of ensuring the dependability of engineering systems is profoundly intricate, extending across all phases of a system's service life. Presently, numerous challenges persist, particularly in sectors such as the construction of maritime power plants, where reliance on logical reasoning remains prevalent.

Numerous scholarly investigations have delved into the realm of reliability systems, aiming to evaluate symbolic reliability expressions tailored for complex systems. Previous scholars [4, 5, 7-12] have contributed significantly to this field.

Gupta and Sharma [2, 3, 6] conducted studies exploring the dependability behavior of power plants,
employing an orthogonalization method. Meanwhile, Gupta and Agarwal [1] specifically tackled the dependability of power supply from a power house to critical consumers. Their analysis operated under the assumption that all cables within the power house were 100% dependable. However, it is essential to note that in practical scenarios, such cables do not necessarily exhibit 100% reliability.

With the aforementioned considerations in mind, the author of this study has examined a comprehensive system consisting of two interconnected subsystems operating in parallel. These subsystems correspond to two power generators, namely P1 and P2, within a power plant setting. Perfectly reliable wires establish connections between generators P1 and P2 and their respective two-way main switches, denoted as TWMS1 and TWMS2. Additionally, these two-way main switches, TWMS1 and TWMS2, are interconnected by cable ‘a3’. Further connections are established through additional cables, ‘a1’ and ‘a2’, linking two-way main switch TWMS1 to the output main switch OPMS3, and two-way main switch TWMS2 to the output main switch OPMS3, respectively.

Consequently, the entire system under consideration can be conceptualized as a power plant, with the primary objective of supplying power generated by P1 and P2 to critical consumers. The reliability of the power supply, originating from output main switch OPMS3, has been estimated utilizing the Boolean function expansion algorithm. This analysis accounts for the fact that failure times for various components of the system follow arbitrary distributions over time. Additionally, an essential parameter known as Mean Time to Failure (MTTF) has been computed for the exponential failure time distribution of various system components.

To elucidate the crucial findings of this study, numerical examples and graphical representations have been incorporated, providing further insight into the reliability assessment of the power plant system.

ASSUMPTIONS

1. The dependability of all system element components is understood in advance.
2. Every components’ states are statistically independent.

![Figure 1. System Configuration](image-url)
3. Each component and the entire system are either in excellent (operational) or bad (failed) condition.
4. No standby or switched redundancy exists.
5. All of the component failure times are arbitrary.
6. There is no repair shop.
7. The system, i.e. the power supply, can fail only if: (i) both generators fail; or (ii) at least one component (switch or cable) in the power supply pathways fails.

The complex system under consideration is shown in Fig. 1.

**NOTATION**

\( \zeta_1, \zeta_2 \) states of subsystems (generators) \( P_1 \) and \( P_2 \)

\( \zeta_3, \zeta_4, \zeta_7 \) states of TWMS1, TWMS2, OPMS3

\( \zeta_5, \zeta_6, \zeta_8 \) states of the cables \( a_1, a_2, a_3 \)

\( \zeta'_k \) Negation of \( \zeta_k \) \( (k = 1 - 8) \)

\( \wedge \) conjunction

\( \lor \) disjunction

\[ \zeta_i = \begin{cases} 0, & \text{in bad state,} \\ 1, & \text{in good state,} \end{cases} \quad (i = 1 - 8) \]

\( Pr(f = 1) \) the probability of the successful operation of the function \( f \).

**DEVELOPMENT OF THE MATHEMATICAL MODEL**

Utilizing the Boolean function technique, the conditions requisite for the successful operation of the intricate system are articulated in the form of logical matrices.

\[
f(\zeta_1, \zeta_2, \ldots, \zeta_8) = \begin{bmatrix} \zeta_1 & \zeta_3 & \zeta_5 & \zeta_7 \\ \zeta_1 & \zeta_3 & \zeta_8 & \zeta_4 & \zeta_6 & \zeta_7 \\ \zeta_2 & \zeta_4 & \zeta_6 & \zeta_7 \\ \zeta_2 & \zeta_4 & \zeta_8 & \zeta_3 & \zeta_5 & \zeta_7 \end{bmatrix}
\]

\( (1) \)

**SOLVING THE MATHEMATICAL FORMULATION**

By the application of algebra of logic equation (1) may be written as

\[
f(\zeta_1, \zeta_2, \ldots, \zeta_8) = \zeta_7 \wedge g(\zeta_1, \zeta_2, \ldots, \zeta_8).
\]

\( (2) \)

where

\[
g(\zeta_1, \zeta_2, \ldots, \zeta_8) = \begin{bmatrix} \zeta_1 & \zeta_3 & \zeta_5 & \zeta_7 \\ \zeta_2 & \zeta_4 & \zeta_6 & \zeta_8 \\ \zeta_3 & \zeta_5 & \zeta_8 \end{bmatrix}
\]

\( (3) \)
Now, with five arguments \((ζ₃, ζ₄, ζ₅, ζ₆, ζ₈)\) entering equation (3) twice, any of them may be selected for the initial expansion. For the sake of illustration, let’s choose \(ζ₈\) and proceed to decompose the complex event into incompatible events as delineated below:

\[
g(ζ₁, ζ₂, \ldots, ζ₈) = ζ₈y₀ \lor ζ₈y₁,
\]  

where

\[
ψ₀ = |ζ₁ ζ₃ ζ₅ | \quad |ζ₂ ζ₄ ζ₆ |
\]  

\[
ψ₁ = |ζ₁ ζ₃ | \quad |ζ₅ ζ₆ | \quad |ζ₁ ζ₄ | \quad |ζ₅ | \quad |ζ₆ |
\]

Since all the letters occur in equation (5) only once, it implies that \(y₀\) is non-iterated. Now, expand the function \(y₁\) by argument \(a₃\) (say) as follows.

\[
ψ₁ = ζ₈y₁₀ \lor ζ₈y₁₁,
\]  

Where

\[
ψ₁₀ = |ζ₂ ζ₄ ζ₆ | \quad \text{(8)}
\]

\[
ψ₁₁ = |ζ₁ | \quad |ζ₃ | \quad |ζ₅ ζ₆ | \quad |ζ₅ ζ₆ | \quad |ζ₃ | \quad |ζ₅ |
\]

Now, the function \(ψ₁₀\) is a non-iterated function while the function \(ψ₁₁\) requires an additional expansion by one of the letters \(ζ₄, ζ₅, ζ₆\). Let us expand \(ψ₁₁\) by argument \(ζ₄\).

\[
ψ₁₁ = ζ₄y₁₁₀ \lor ζ₄y₁₁₁,
\]  

Where

\[
ψ₁₁₀ = |ζ₁ ζ₅ | \quad \text{(11)}
\]

\[
ψ₁₁₁ = |ζ₁ ζ₅ | \quad |ζ₂ | \quad |ζ₆ |
\]

Now, all the functions are non-iterated and are not subjected to further transformations, so, making
use of equations (4)—(12), we get

\[
g(\zeta_1, \zeta_2, \ldots, \zeta_8) = \begin{vmatrix}
\zeta_8' & \zeta_1 & \zeta_3 & \zeta_5 \\
\zeta_2 & \zeta_4 & \zeta_6 \\
\zeta_3 & \zeta_4 & \zeta_1 & \zeta_5 \\
\zeta_4 & \zeta_1 & \zeta_5 & \zeta_2 \\
\zeta_5 & \zeta_3 & \zeta_6 & \zeta_4 \\
\zeta_6 & \zeta_8 & \zeta_2 & \zeta_3 \\
\zeta_7 & \zeta_9 & \zeta_4 & \zeta_5 \\
\zeta_8 & \zeta_7 & \zeta_9 & \zeta_4 \\
\zeta_9 & \zeta_8 & \zeta_7 & \zeta_5 \\
\end{vmatrix}
\]

(13)

Equation (13) may be written as

\[
g(\zeta_1, \zeta_2, \ldots, \zeta_8) = \begin{vmatrix}
H_1 & \zeta_1 & \zeta_3 & \zeta_5 \\
H_2 & \zeta_2 & \zeta_4 & \zeta_6 \\
H_3 & \zeta_3 & \zeta_1 & \zeta_5 \\
H_4 & \zeta_4 & \zeta_5 & \zeta_2 \\
\end{vmatrix}
\]

(14)

where

\[
H_1 = \zeta_8' \\
H_2 = \zeta_8' \zeta_3' \\
H_3 = \zeta_8' \zeta_3' \zeta_4' \\
H_4 = \zeta_8' \zeta_3' \zeta_4' \\
\]

(15) to (18)

Now using Bayes' formula, the probability of successful operation of the function \(g\) is given by

\[
Pr(g = 1) = Pr(H_1) Pr(g/H_1) + Pr(H_2) Pr(g/H_2) + Pr(H_3) Pr(g/H_3) + Pr(H_4) Pr(g/H_4) = Pr(\zeta_8') Pr(\psi_0) + Pr(\zeta_8' \zeta_3') Pr(\psi_10) + Pr(\zeta_8' \zeta_3' \zeta_4') Pr(\psi_{110}) + Pr(\zeta_8' \zeta_3' \zeta_4') Pr(\psi_{111})
\]

(19)

If \(Z_i\) is the reliability of the component of the complex system corresponding to state \(a_i\) and \(Q_i\) is the corresponding unreliability, then from equation (19)

we get

\[
Pr(g = 1) = Q_8[1 - (1 - Z_1 Z_3 Z_5)(1 - Z_2 Z_4 Z_6)] + Z_8 Q_3 Z_2 Z_4 Z_6 + Z_8 Z_3 Q_4 Z_1 Z_2 + Z_8 Z_2 Z_4 (1 - Q_1 Q_2) (1 - Q_5 Q_6)
\]

(20)

Finally, the probability of the successful operation (i.e. reliability) of the complex system is given by

\[
Z_s = Pr(f = 1) = Pr(C_7). Pr(g = 1) = 2Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 - Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 \\
- Z_8 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 - Z_1 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 \\
- Z_8 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 - Z_1 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 \\
+ Z_1 Z_3 Z_4 Z_5 Z_6 Z_7 + Z_2 Z_3 Z_4 Z_5 Z_7 Z_8 \\
+ Z_1 Z_3 Z_5 Z_7 + Z_2 Z_4 Z_6 Z_7.
\]

(21)
SPECIFIC INSTANCES

Case I

If the reliability of each component of the complex system is \( Z \), equation (21) yields

\[
Z_s = Z^4(2Z^4 - 5Z^3 + 2Z^2 + 2).
\] (22)

Case II. When Failure Rates Follow Weibull Distribution

The complex system's reliability at a specific moment \( t \) can be derived using equation (21). Denoting the failure rates of the sub-systems (generators) \( P_1, P_2, \) TWMS1, TWMS2, OPMS3, and cables \( a_1, a_2, a_3 \) as \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \) and \( \alpha_7, \) respectively.

\[
Z_{sw}(t) = 2 \exp(-j_1 t^p) - \exp(-j_2 t^p) - \exp(-j_3 t^p) \]
\[
- \exp(-j_4 t^p) - \exp(-j_5 t^p) - \exp(-j_6 t^p) + \exp(-j_7 t^p) + \exp(-j_8 t^p) + \exp(-j_9 t^p) + \exp(-j_{10} t^p)
\] (23)

where \( p \) is a positive parameter and \( j_i \) are given by

\[
j_1 = A = \sum_{i=1}^{8} \alpha_i,
\]
\[
j_2 = A - \alpha_8,
\]
\[
j_3 = A - \alpha_5,
\]
\[
j_4 = A - \alpha_6,
\]
\[
j_5 = A - \alpha_2,
\]
\[
j_6 = A - \alpha_3,
\]
\[
j_7 = A - (\alpha_2 + \alpha_5),
\]
\[
j_8 = A - (\alpha_1 + \alpha_6),
\]
\[
j_9 = \alpha_1 + \alpha_3 + \alpha_5 + \alpha_7,
\]
\[
j_{10} = \alpha_2 + \alpha_4 + \alpha_6 + \alpha_7.
\]

Table 1. System Reliability Table

<table>
<thead>
<tr>
<th>Serial number</th>
<th>Time t</th>
<th>System reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( Z_{se} (t) )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.42871</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.11064</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0.02537</td>
</tr>
</tbody>
</table>
Figure 2. System Reliability Vs. Time

Figure 3. System Reliability Vs. Time in 3D Surface
Case III. When Failure Rates Follow Exponential Distribution

Given that the exponential distribution serves as a specific instance of the Weibull distribution for \( p = 1 \), it holds substantial utility across numerous real-world scenarios. In this context, the reliability of the complex system at any given moment \( t \) is influenced by

\[
Z_{SE}(t) = [Z_{SW}(t)]_{at \ p=1} = 2 \exp(-j_1t) - \exp(-j_2t) - \exp(-j_3t) - \exp(-j_4t) - \exp(-j_5t) - \exp(-j_6t) + \exp(-j_7t) + \exp(-j_8t) + \exp(-j_9t) + \exp(-j_{10}t).
\] (24)

The expression for M.T.T.F. in this case is given by

\[
\text{M.T.T.F.} = \int_0^\infty Z_{SE}(t)dt = \frac{2}{J_1} - \frac{1}{J_2} - \frac{1}{J_3} - \frac{1}{J_4} - \frac{1}{J_5} + \frac{1}{J_6} + \frac{1}{J_7} + \frac{1}{J_8} + \frac{1}{J_{10}}.
\] (25)

Table 2. Mean Time to Failure Table

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>( \alpha )</th>
<th>M.T.T.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>( \infty )</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>1.84524</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.92262</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>0.61508</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>0.46131</td>
</tr>
</tbody>
</table>

Figure 4. Failure Rate Vs. M.T.T.F.
MATHEMATICAL EVALUATION OF RELIABILITY

Setting $\alpha_i = 0.3$, $\alpha_2 = 0.4$, $\alpha_3 = \alpha_4 = \alpha_7 = 0.1$, $\alpha_5 = \alpha_6 = \alpha_8 = 0.2$ and $p = 2$ in equations (23) and (24), we get Table 1.

CONCLUSION

Assuming either an exponential or Weibull distribution for failures, Table 1 illustrates the system's dependability at various time intervals. Notably, the reliability of the complex system exhibits a gradual decline at a uniform rate when failure follows an exponential distribution. However, when
failure adheres to a Weibull distribution, the system's reliability diminishes at an exceedingly rapid pace, as highlighted in a comprehensive analysis of the reliability versus time graph (Fig. 2).

Furthermore, Table 2 presents calculations for the average time to system failure across different failure rate values. Upon scrutinizing the graph depicting Mean Time to Failure (MTTF) versus failure rate (Fig. 3), it becomes apparent that MTTF initially experiences a significant decline, followed by a more gradual descent. Consequently, this analysis facilitates the computation of the mean time to system failure and the dependability of power supply for a complex system, based on a specified set of component failure rates.

REFERENCES